## D-5316

M.A./M.Sc. ( ${ }^{\text {st }}$ Semester)

Examination, 2020

## MATHEMATICS

(Advanced Discrete Mathematics - I)
Time Allowed : Three Hours
Maximum Marks : 70
Note : Attempt all sections as per instruction given.

## SECTION -A

(Objective Type Question)
Q. 1. Choose the correct option : $\mathbf{2 \times 5 = 1 0}$
(i) An element $x$ in a semigroup ( $\mathrm{S}, *$ ) is called an idempotent if :
(a) $x=x$
(b) $x+x=x$
(c) $\mathrm{x}^{2}=\mathrm{x}$
(d) $x=1$
(ii) Evey finite lattice is:
(a) Partially bounded
(b) Unbounded
(c) Bounded
(d) None of these
(iii) In a Boolean algebra $B$ if $a+x=b+x$ and $\mathrm{a}+\mathrm{x}^{\prime}=\mathrm{b}+\mathrm{x}^{\prime}$ then :
(a) $a=a^{\prime}$
(b) $\mathrm{a}^{\prime}=\mathrm{b}$
(c) $a=b$
(d) $a=x^{\prime}$
(3)
(iv) The set of language generated by regular grammar is called :
(a) Phase structure grammar
(b) Production rule
(c) Regula set
(d) None of these
(v) An elements $a$ and $b$ of a Boolean algebra B then it is called Demorgan's law :
(a) $(a+b)^{\prime}=a^{\prime} b^{\prime}$
(b) (ab)' $=a^{\prime}+b^{\prime}$
(c) Both (a) and (b)
(d) None of these

## (4)

## SECTION - B

## (Very Short Answer Type Questions)

Q. 2. Answer the following questions in two or three sentences each (attempt any five) : $\quad \mathbf{5 \times 2 = 1 0}$
(i) What is semigroup?
(ii) Define Automorphism.
(iii) What is phrase structure grammar?
(iv) Define Regular grammar.
(v) Define Boolean function.
(vi) Write a Demorgan law for Boolean algebra.
(vii) Define isomorphism of semigroup.
(5)
(6)

## SECTION - C

## (Short Answer Type Questions)

Note : Attempt any 5 questions.
$5 \times 4=20$
Q. 3. (i) Prove that the statement $(p \vee q) \Leftrightarrow(\sim p \wedge \sim q)$
is a tautology.
(ii) Discuss absorption law for Boolean algebra.
(iii) Construct a grammar for the language $L=\left\{a^{i} b^{2 i}: i \geq 1\right\}$.
(iv) Define Lattice give an example.
(v) Change the following functions to disjuctive normal forms of three variables $\mathrm{x}, \mathrm{y}, \mathrm{z}$ :
(a) $x+y^{\prime}$
(b) $x^{\prime} z+x z^{\prime}$
(vi) Prove that every semigroup has an idempotent element.
(vii) Express the three connective $\mathrm{v}, \wedge$ and $\sim$ in terms of the connective $\perp$.

## SECTION - D

## (Long Answer Type Questions)

Note : Attempt any three questions.
Q. 4. (i) Show that dual of a lattice is a lattice.
(ii) State and prove fundamental theorem of homomorphism of semigroups.

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(iii) Draw a switchen circuit and simplified
switchen circuit of the following expression :
$F(x, y, z)=x y^{\prime} z+(z+y) x^{\prime}$ verify it.
(iv) Define grammar and language over grammar, consider the grammar $G=(N, T$, $P, S)$ where $N=\{S, A\}, T=(a, b)$ and $P=$ $\{S \rightarrow a A, S \rightarrow b, A \rightarrow a a\}$ with start symbol
S. Find $L(G)$.

