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D-5332

M.Sc. (III[rd] Semester) Examination, 2020

MATHEMATICS

(Partial Differential Equations and Mechanics - I)

Time Allowed : Three Hours

Maximum Marks : 70

Note : Attempt questions from all four sections as directed. Distribution of marks is given with each section.

SECTION - A

Note : Attempt all questions of this section. Each question carries one mark. **10×1=10**

Q. 1. Fill in the blanks type questions :

(i) The general theory of solutions to Laplace's equation is known as _______.

(Potential theory / Laplacian operator theory)

(ii) Method to find the solution of a PDE by converting it into ODE is called _______.

(Method of characteristics / energy method)

(iii) $u_t - \Delta u = 0$ is called _______.

(the heat equation / the wave equation)

(iv) $u_{tt} - \Delta u = 0$ is called _______.

(the heat equation / the wave equation)

(v) $\nabla^2 V = 0$ is known as _______, where V is the potential of the system of attracting particles.

(Poisson's equation for potential / Laplace equation for potential)

Multiple choice type questions.

Choose the correct alternative :

(vi) Possible solution of equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is :

(a) $u = (c_1 \cos px + c_2 \sin px)(c_3 e^{py} + c_4 e^{-py})$

(b) $u = (c_1 e^{px} + c_2 e^{-px})(c_3 \cos py + c_4 \sin py)$

(c) $u = (c_1 x + c_2)(c_3 y + c_4)$

(d) All of these

(vii) One dimensional wave equation is :

(a) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

(b) $\frac{\partial^2 u}{\partial t^2} = -c^2 \frac{\partial^2 u}{\partial x^2}$

(c) $\frac{\partial^2 u}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial x^2}$

(d) $\frac{\partial^2 u}{\partial t^2} = -\frac{1}{c^2} \frac{\partial^2 u}{\partial x^2}$

(viii) Poisson equation is an example of :

(a) Elliptic PDE

(b) Hyperbolic PDE

(c) Parabolic PDE

(d) None of these

(ix) If a point is inside the spherical shell of radius a and mass M then the attractive at that point will be :

(a) 0

(b) $\frac{M}{a^2}$

(c) $\frac{M}{2a^2}$

(d) None of these

(x) Attraction at any point P on the axis of a uniform circular disc of infinite radius is :

(a) 0

(b) $\frac{2M}{a^2}$

(c) $\frac{M}{a^2}$

(d) None of these

SECTION - B

Note : Attempt any five questions. Each question carries 2 marks. **5×2=10**

Q. 2. Very short answer type questions (25-30 words) :

(i) What is Laplace equation ? What is it used for ?

(ii) Define Legendre transform.

(iii) Solve PDE yzp + zxq = xy.

(iv) What is Euler-Lagrange equation ?

(v) What is variational principle ?

(vi) What is method of characteristics ?

(vii) What is a Riemann's problem ? Where are they useful ?

SECTION - C

Note : Attempt any five questions. Each question carries 4 marks. **5×4=20**

Q. 3. Short answer type questions (250 words) :

(i) Discuss physical interpretation of Laplace equation.

(ii) Find mean value formula for heat equation.

(iii) Solve using characteristics :

$x_1 u_{x_1} + x_2 u_{x_2} = 2u, \; u(x_1,1) = g(x_1)$

(iv) Find a function to satisfy transport equation $u_t + cu_x = 0$ and initial condition $u(x, 0) = f(x)$. C is a fixed constant.

(v) Find the solution of IVP

$$\begin{cases} u_t + b\,Du = 0 & \text{in} \quad R^n \times (0,\infty) \\ u = g & \text{on} \quad R^n \times \{t = 0\} \end{cases}$$

Here $b \in R^n$ and $g : R^n \to R$ are known.

(vi) Find the potential at an external point due to a uniform straight rod.

(vii) Find Poisson's equation for potential of a system of attracting particles.

SECTION - D

Note : Attempt any three questions. Each question carries 10 marks. **3×10=30**

Q. 4. Essay type questions (more than 500 words) :

(i) Find the fundamental solution of the heat equation $u_t - \Delta u = 0$.

(ii) Prove that $u(x, t) = G\left(\frac{x - y(x,t)}{t}\right)$ for a.e.x. in an integral solution of I.V.P. for scalar conservation laws in one space dimension :

$$\begin{cases} u_t + F(u)_x = 0 & \text{in} \quad R \times (0,\infty) \\ u = g & \text{on} \quad R \times \{t = 0\} \end{cases}$$

Here $F : R \to R$ and $g : R \to R$ are given and $u : R \times (0, \infty) \to R$ is unknown, $u = u(x, t)$.

(iii) Find attraction at any point on the axis of a uniform circular disc.

(iv) Find the attraction of a spherical shell of radius a at a point P at a distance r from the centre O of the shell.