# D-5312

## M.Sc. (Ist Semester) Examination, 2020

## MATHEMATICS

### Paper - I

### (Real Analysis)

***Time Allowed : Three Hours***

***Maximum Marks : 70***

**Note :** Attempt questions of all four sections as directed.

### SECTION - A

**Note :** Attempt all 10 questions. Each question carries one mark. **1×10=10**

**Q. 1.** Objective Type :

(i) If $f \in R(\alpha)$ on [a, b] and if a < c < b then $f \in R(\alpha)$ on [a, c] and on [c, b] and :

(a) $\int_a^c f\, d\alpha + \int_c^b f\, d\alpha = \int_a^b f\, d\alpha$

(b) $\int_a^b f_1\, d\alpha + \int_b^c f_2\, d\alpha = \int_a^b f\, d\alpha$

(c) $\int_0^a f\, d\alpha + \int_a^b f\, d\alpha = \int_a^b f\, d\alpha$

(d) None





(ii) A curve $\gamma$ is said to be Rectifiable, if arc length of is $\Delta_\gamma$ [a, b] :

(a) $\Delta_\gamma(a,b) \le \int_a^b |r'(t)| dt$

(b) $\Delta_\gamma(a,b) \le \inf \Delta_\gamma(P)$

(c) $\Delta_\gamma(a,b) = \sup \Delta_\gamma(P)$

(d) None

(iii) Value of series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots\ldots\ldots\ldots$ :

(a) log2

(b) $\frac{1}{2}$ log2

(c) $\frac{3}{2}$ log2

(d) 0

(iv) We can test uniform convergence by the test :

(a) Mn-test

(b) Riemann integral

(c) Weiestrass approximation

(d) None

(v) A pt. of non-uniformly convergence of sequence $<f_n(x)>$, where $f_n(x) = nxe^{-nx^2}, x \in R$ :

(a) 1

(b) Does not exist

(c) 0

(d) None

(vi) $\sum \frac{1}{n^p + n^q x^2}$ is uniformly convergent for all values of x if :

(a) $p + q < 1$

(b) $p + q = 1$

(c) $p + q > 2$

(d) $p + q = 0$

(vii) The totality of pts X for which a power series converges is called :

(a) Radius of convergence

(b) Region of convergence

(c) Region of divergence

(d) None





(viii) Series $\sum_{n=0}^{\infty} \frac{x^n}{n}$ converges for :

(a) $-1 \le x < 1$

(b) $-1 < x < 1$

(c) $0 < x < 1$

(d) $0 < x \le 1$

(ix) $\lim_{(x,y)\to(0,0)} f(x,y)$, where $f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$ :

(a) 1

(b) 0

(c) 1/2

(d) Does not exist

(x) If u is homogeneous function of x and y of degree n then :

(a) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$

(b) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n$

(c) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = f(u)$

(d) None

## SECTION - B

**Note :** Attempt any five questions.

**Q. 2.** Very short answer type : **5×2=10**

(i) If f be RS-integrable function on [a, b], then |f| is also RS-integrable on [a, b] then show

$$\left|\int_a^b f\, d\alpha\right| \le \int_a^b |f|\, d\alpha.$$

(ii) Show the series $\Sigma u_n(x)$ converges on X if and only if for every $\in > 0\ \exists$ a positive integer m s.t.

$n \ge m \Rightarrow |u_{n+1}(x) + u_{n+2}(x) + \ldots\ldots + u_{n+p}(x)| < \in$

p = 1, 2, 3 ........

$\forall\ x \in X$.

(iii) Show that sequence $<f_n>$, where

$f_n(x) = \dfrac{nx}{1+n^2x^2}$, does not converge uniformly on R.

(iv) Examine for term by term integration, the series for which $f_n(x) = nxe^{-nx^2}$, $x \in R$. Also indicate the interval for which your conclusion holds.





(v) Show that the series :

$$\sum_{n=1}^{\infty} \frac{1}{n^3 + n^4x^2},\ x \in R$$

is uniformly convergent for all values of x and is differentiable term by term.

(vi) Find Radius of convergence of power series :

$$\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$$

(vii) State and prove Able's theorem.

## SECTION - C

**Note :** Attempt any five questions. **5×4=20**

**Q. 3.** Short answer type questions :

(i) If $f \in RS(\alpha)$ on [a, b] for $a \le x \le b$

put $F(x) = \int_a^b f(t)\, d\alpha$ then F is continuous on [a, b]. Also if f is continuous at pt. $x_0$ of [a, b], then F is differentiable at $x_0$ and $F'(x_0) = f(x_0)$.

(ii) State and prove fundamental theorem of integral calculas.

(iii) State and prove Mn-test for uniform convergence.

(iv) The sum of an absolute convergent series does not alter with any rearrangement of term.

(v) State and prove general principle of uniform convergence.

(vi) If E be an open set in $R^n$, f maps into $R^m$, f be differentiable at $x_0 \in E$, g maps an open set containing f(E) into $R^k$ and g be differentiable at $f(x_0)$ then mapping F of E into $R^k$ defined by F(x) = g f(x) is differentiable at $x_0$ and $F'(x_0) = g'(f(x_0))\, f'(x_0)$.

(vii) Find the rectangle of perimeter which has maximum area.

**SECTION - D**

**Note :** Attempt any three questions. **3×10=30**

**Q. 4.** (i) If $\gamma$ be continuously differentiable curve on [a, b] then $\gamma$ is rectifiable and





$$\Delta_\gamma \int_a^b |\gamma'(t)|dt$$

If $\gamma'$ is continuous on [a, b] then $\gamma$ is rectifiable and $\Delta(r) = \int_a^b |\gamma'(t)|dt.$

(ii) State and prove Riemann's theorem.

(iii) State and prove Weirstrass's approximation theorem.

(iv) State and prove Implicit function theorem.

(v) Find the maximum and minimum value of function

$$f(x, y, z) = x^2 + y^2 + z^2$$

subject to constraints

$x^2 + y^2 + z^2 + xy + yz + zx = 1$ and

$x + 2y - 3z = 0$