D-5345

M.A./M.Sc. (IVth Semester) Examination, 2020

MATHEMATICS

(Numerical Analysis - II)

Time Allowed : Three Hours

Maximum Marks : 70

SECTION - A

Note : Attempt all ten questions. Each question carries one mark. **10**

Q. 1. Objective type :

(i) The difference equation $\Delta y_k - 2y_k = 3$ can be written in subscript notation as :

(a) $y_{k+1} - 3y_k = 0$

(b) $y_{k+2} - 2y_k = 0$

(c) $y_{k+1} + 2y_k = 3$

(d) $y_{k+1} - 3y_k = 3$

(ii) The solution of the linear difference equation $y_{k+1} - ay_k = 0,\ a \neq 1$ is :

(a) $y_k = ca^k$

(b) $y_{k+1} = ca^{k+1}$

(c) $y = ca^k$

(d) $y_{k+1} = ca^{k+1}$

(iii) The order and degree of the difference equation $y_{k+2} - 2y_{k+1} = y_k = c$ is :

(a) 2, 1

(b) 0, 2

(c) 2, 2

(d) 1, 1

(iv) The generating function of the sequence 0, 1, 2, 3, ... ∞ is :

(a) $t(1-t)^{-3}$

(b) $(1-t)^{-1}$

(c) $(1-t)^{-2}$

(d) $t(1-t)^{-2}$

(v) Newton's iterative formula for obtaining a^{-1} is :

(a) $x_{n+1} = x_n (2 - ax_n)$

(b) $x_{n+1} = x_n (1 - ax_n)$

(c) $x_{n+1} = x_n (2 + ax_n)$

(d) $x_{n+1} = x_n (1 + ax_n)$

(vi) One root of the equation $x^3 - x - 1 = 0$ lies between :

(a) 0 and 1

(b) 2 and 3

(c) 1 and 2

(d) None of these

(vii) In Runge-Kutta method, value of k_2 is :

(a) $hf(x_0, y_0)$

(b) $hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_3}{2}\right)$

(c) $hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

(d) $hf\left(x_0 - \frac{h}{2}, y_0 - \frac{k_3}{2}\right)$

(viii) The solution of simultaneous linear algebraic equation can not be found by :

(a) Gauss-Seidel method

(b) Gauss-Jordan method

(c) Gauss-Elimination method

(d) None of these

(ix) Which of the following is a direct method :

(a) Relaxation method

(b) Gauss-Seidel method

(c) Crout's method

(d) None of these

(x) Relaxation method converges __________ in comparison to Gauss-Seidel iterative method :

(a) Less rapidly

(b) Remains constant

(c) More rapidly

(d) None of these

SECTION - B

Note : Attempt any five questions. Each question carries 2 marks. **10**

Q. 2. Very short answer type (25-30 words) :

(i) Solve the difference equation :

$y_{k+2} - 2y_{k+1} + 5y_k = 0$

(ii) Write the types of iterative method.

(iii) Write the method of iteration.

(iv) Find $\frac{1}{E^2 - 6E + 8}\left(3k^2 + 2\right)$.

(v) Solve the difference equation :

$9y_{k+2} - 6y_{k+1} + y_k = 0$

(vi) The equation $x^6 - x^4 - x^3 - 1 = 0$ has one real root between (1.4, 1.5), find this root to three decimal places by false position method.

(vii) Solve x = 0.21 sin (0.5 + x) by iteration method starting with $x_0 = 0.12$.

SECTION - C

Note : Attempt any five questions. Each question carries 4 marks. **20**

Q. 3. Short answer type (250 words) :

(i) Solve :

$y_{k+2} - 3y_{k+1} + 2y_k = 2^k$

(ii) Use Picard's method to approximate the value of y when x = 0.1, given that y = 1 when x = 0 and

$$\frac{dy}{dx} = \frac{y - x}{y + x}$$

(iii) Find a root of the equation $f(x) = x^3 - 4x - 9 = 0$ using the bisection method in four iterations.

(iv) Use Runge's method to approximate y when x = 1.1, given that y = 1.2 at x = 1 and

$$\frac{dy}{dx} = 3x + y^2$$

(v) Use Euler's method with h = 0.1 to find the solution of $\frac{dy}{dx} = x^2 + y^2, y(0) = 0$ in the range $0 \le x \le 0.5$.

(vi) Solve by Gauss's elimination method the following :

$$6x + 3y + 2z = 6$$
$$6x + 4y + 3z = 0$$
$$20x + 15y + 12z = 0$$

(vii) Obtain $\sqrt{12}$ to five places of decimals by Newton's Raphson method.

SECTION - D

Note : Attempt any three questions. Each question carries 10 marks. **30**

Q. 4. Essay type (more than 500 words) :

(i) Solve the following equations by Crout's method :

$$x + 2y + 3z = 6$$
$$2x + 3y + z = 9$$
$$3x + y + 2z = 8$$

(ii) Solve by Jacobi method :

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

(iii) Find y(2) if y(x) is the solution of $\frac{dy}{dx} = \frac{x+y}{2}$, assuming y(0) = 2, y(0.2) = 2.636, y(1.0) = 3.595 and y(1.5) = 4.968 using Milne's method.

(iv) Using Picard's method of successive approximation, obtain a solution upto the fifth approximation of the equation :

$$\frac{dy}{dx} = y + x, \ y(0) = 1$$