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M-5343

M.A./M.Sc. (IVth Semester)

Examination, 2020

MATHEMATICS

(Complex Variable)

Time Allowed : Three Hours

Maximum Marks : 70

Note : Attempt any five questions. All questions carry equal marks.

Q. 1. State and prove Cauchy-Gaursat theorem.

Q. 2. Evaluate using Cauchy integral formula :

(i) $\int_C \frac{e^z}{(z+1)^2} dz$

where C is the circle |z – 1| = 3

(ii) $\int_C \frac{\log z}{(z-1)^z} dz$

where C is the circle $|z-1| = \frac{1}{2}$.

Q. 3. State and prove Taylor's series theorem.

Q. 4. Find the kind of the singularity of the following function :

(i) $f(z) = \frac{\cot \pi z}{(z-a)^2}$

(ii) $\sin \frac{1}{1-z}$

(iii) $\operatorname{cosec} \frac{1}{z}$

Q. 5. State and prove Rouche's theorem.

Q. 6. (a) By using Cauchy residue theorem prove :

$$\int_0^\pi \frac{d\theta}{a + \cos\theta} = \frac{\pi}{\sqrt{a^2 - 1}} \quad a > 1$$

(b) By using Cauchy residue theorem prove :

$$\int_0^\infty \frac{\cos mx}{a^2 + x^2} dx = \frac{\pi}{2a} e^{-ma}$$

where $m \geq 0$, $a > 0$

Q. 7. State and prove Runge's theorem.

Q. 8. If Re(z) > 1 then prove that :

$$\zeta(z)\Gamma z=\int_0^{\infty}\frac{t^{z-1}}{\left(e^{t-1}\right)}dt$$

Q. 9. (a) Show that the series $\sum_{n=0}^{\infty}\frac{z^n}{2n+1}$ and $\sum_{n=0}^{\infty}\frac{(z-i)^n}{(2-i)^{n+1}}$ are analytic continuation of each other.

(b) Prove that an analytic function can not be more than one continuation into the same domain.

Q. 10. State and prove monodromy theorem.