Printed Pages – 7

D-5341

M.A./M.Sc. (IVth Semester) Examination, 2020

MATHEMATICS

Paper - I

(Integration Theory & Functional Analysis - II)

Time Allowed : Three Hours

Maximum Marks : 70

SECTION - A

Note : Attempt all ten questions. Each question carries one mark. **10**

Q. 1. Objective Type :

(i) Let B and B' be two banach spaces and if T is continuous linear transformation of B onto B' then T is an ______.

(ii) E is a projection on M along N if and only if I–E is a ______ on N along M.

(iii) If X and Y are normed linear space and D be a subspace of X and T : D $\rightarrow$ Y then ______.

(iv) An inner product space is also called a ______.

(v) The orthogonal compliment of a subset of a Hilbert space is ______.

(vi) Let M and N be closed linear subspaces of a Hilbert space H such that M $\perp$ N, then the linear subspace M + N is also :

(a) Open

(b) Closed

(c) Bounded

(d) None of these

(vii) An orthonormal set can not contain :

(a) Unit vector

(b) Zero vector

(c) Zero

(d) Unit

(viii) A linear operator T on a Hilbert space H is said to be self adjoint is :

(a) $T^* = T$

(b) $T^{**} = T$

(c) $T_x = T_y^*$

(d) All of above

(ix) Identity and zero operators are both :

(a) Normal operator

(b) Unitary operator

(c) Positive operators

(d) All the above

(x) If N is a normal operator on a Hilbert space H, then :

(a) $N^2 = N$

(b) $||N^2|| = ||N||^2$

(c) $|N^2| = |N|^2$

(d) None of these

SECTION - B

Note : Attempt any five questions. Each question carries two marks. **10**

Q. 2. Very short answer type (25-30 words) :

(i) Let H be a Hilbert space, an operator T on H is said to be normal if and only if $\|T_x^*\| = \|T_x\|$.

(ii) An operator T on a Hilbert space H is self-adjoint if and only if (T_x, x) is real for all X.

(iii) The adjoint operator $T \to T^*$ on B(H) then prove that $(T_1 + T_2)^* = T_1^* + T_2^*$.

(iv) Let H be a Hilbert space. Then prove that the inner product is jointly continuous.

(v) Define closed linear transformation.

(vi) Define Hilbert space.

(vii) Let S be a non-empty subset of a Hilbert space H, then show that $S^{\perp} = S^{\perp\perp\perp}$.

SECTION - C

Note : Attempt any five questions. Each question carries four marks. **20**

Q. 3. Short answer type (250 words) :

(i) Write and prove projection theorem for Hilbert space.

(ii) Write and prove Schwarz Inequality for Hilbert space.

(iii) Let H be a Hilbert space and M be a linear subspace of H. Then M is closed if and only if $M = M^{\perp\perp}$.

(iv) T is a closed linear transformation if and only if its graph T_G is a closed subspace.

(v) Write and prove Bessel's inequality for finite orthonormal set.

(vi) Prove that the adjoint operator is one-one onto as a mapping of B(H) into itself.

(vii) If T is an arbitrary operator on Hilbert space H and α, β are scalars such that $|\alpha| = |\beta|$. Then prove that $\alpha T + \beta T^*$ is normal.

SECTION - D

Note : Attempt any three questions. Each question carries ten marks. **30**

Q. 4. Essay type (more than 500 words) :

(i) Write and prove open mapping theorem.

(ii) Write and prove Riesz representation theorem.

(iii) Write and prove Hahn-Banach theorem for linear spaces.

(iv) Let B be a complex Banach space whose norm obeys the parallelogram law and polarisation identity, i.e.

$$4(x, y) = ||xy||^2 - ||x - y||^2 + i||x + iy||^2 - i||x - iy||^2$$

then B is a Hilbert space.