Printed Pages - 7 (2) **D-5342** (iv) The relation between Poisson bracket and M.A. / M.Sc. (IVth Semester) Examination, 2020 Lagrange bracket is _____. **MATHEMATICS** (v) Hamiltonian canonical equations of motion (Partial Differential Equations and Mechanics - II) for a conservative system are _____. Time Allowed : Three Hours Multiple choice type questions : Maximum Marks : 70 (vi) Which of the following true for Poisson brackets ? Note : Attempt questions from all four sections as (a) They follow distributive law directed. Distribution of marks is given with each They follow commutative law (b) section. (c) Both (a) and (b) are true **SECTION - A** (d) None of the above Note: Attempt all questions of this section. Each (vii) The Legendre transform of L, L*(P) is given question carries one mark. 10×1=10 by : (a) $\sup_{q\in R^n} \left\{ p \cdot q + L(q) \right\}$ **Q. 1.** Fill in the blanks type questions : (i) $U_t + U_{xxx} = 0$ is called _____. (b) $\sup_{q\in R^n} \left\{ p \cdot q - L(q) \right\}$ (ii) Hamiltonian H is defined for _____. (c) $\inf_{q \in \mathbb{R}^n} \left\{ p \cdot q + L(q) \right\}$ (iii) The first form of Jacobi's theorem states that (d) $\inf_{q \in \mathbb{R}^n} \left\{ p \cdot q - L(q) \right\}$

D-5342

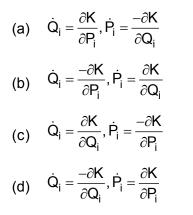
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D-5342

(3)

(viii) $\frac{-d}{ds}(D_p)$	$L(\dot{X}(s), X(s)) + D_{x}L(s)$	$(\dot{X}(s), X(s)) = 0$
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- $\forall s \in [0, t]$ is called :
- (a) Euler-Lagrange equation
- (b) Hamilton-Jacobi equation
- (c) Jacobi equation
- (d) None of the above
- (ix) The following is called wave equation :
 - (a) $|D_U| = 1$
 - (b) $U_t + UU_x = 0$
 - (c) $U_{tt} \Delta U = 0$
 - (d) $U_t \Delta U = 0$
- (x) In a canonical transformation :



(4)

- (xi) Out of the two statements :
 - (i) Lagrange's brackets are invariant under

canonical transformation

(ii) Poisson brackets are invariant under

canonical transformation

- (a) (i) is true, (ii) is false
- (b) (i) is false, (ii) is true
- (c) (i) and (ii) both are true
- (d) (i) and (ii) both are false

(xii) If U, V $\in L^2(\mathbb{R}^n)$, then which one of will hold ?

- (a) $\int_{\mathbb{R}^n} U\overline{V} dx = \int_{\mathbb{R}^n} \hat{U}\overline{V} dy$
- (b) $D^{\alpha}U = (iy)^{\alpha} \dot{U}$ for each multi index α
- (c) $(\hat{U})^{v} = U$
- (d) All of the above

D-5342

P.T.O.

D-5342

SECTION - B

Note : Attempt any five questions. Each question carries

2 marks. 5×2=10

- **Q. 2.** Very short answer type questions (25-30 words) :
 - (i) Define Hamilton canonical equations.
 - (ii) State the second form of Jacobi theorem.
 - (iii) Explain Laplace method.
 - (iv) Define Potential functions.
 - (v) Explain Non-holonomic system.
 - (vi) Explain Poisson's bracket.
 - (vii) Define plane and travelling wave under similarity solution.

SECTION - C

Note : Attempt any five questions. Each question carries

4 marks. 5×4=20

- Q. 3. Short answer type questions (250 words) :
- D-5342 P.T.O.

(6)

- Derive Hamilton principle form Lagrange's equation.
- (ii) State and prove first form of Jacobi theorem.
- (iii) Verify that the following transformation is

canonical transformation :

$$\mathsf{P} = \frac{1}{2} (\mathsf{p}^2 + \mathsf{q}^2), \mathsf{Q} = \tan^{-1} \left(\frac{\mathsf{q}}{\mathsf{p}}\right)$$

- (iv) Write short notes of Hodograph transforms.
- (v) Derive stationary phase for wave equation.
- (vi) Prove that Lagrange's brackets are invariant

under canonical transformation.

(vii) State and prove properties of Fourier

transform.

SECTION - D

Note : Attempt	any	three	questions.	Each	question
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carries 10 marks. 3×10=30

D-5342

(7)

- Q. 4. Essay type questions (more than 500 words) :
 - (i) Apply separation of variable technique to discover a solution of the porous medium equation $U_t - \Delta(U^r) = 0$ in $\mathbb{R}^n X$ (0, ∞) where $U \ge 0$ and r > 1 constant.
 - (ii) State and prove Donkin's theorem.
 - (iii) Using Hopf-Cole transform, solve the quasilinear parabolic equation :

 $\textbf{U}_{t}-\textbf{a}\;\Delta\;\textbf{U}+\textbf{b}\;|\textbf{D}\textbf{U}|^{2}$ = 0 in \textbf{R}^{4} × (0, $\infty)$ and

- $U = g \text{ on } R^4 \times \{t = 0\}$
- (iv) Explain : "The Euler's dynamical equation for

the motion of a rigid body about an axis."