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D-5342

M.A. / M.Sc. (IVth Semester) Examination, 2020

MATHEMATICS

(Partial Differential Equations and Mechanics - II)

Time Allowed : Three Hours

Maximum Marks : 70

Note : Attempt questions from all four sections as directed. Distribution of marks is given with each section.

SECTION - A

Note : Attempt all questions of this section. Each question carries one mark. **10×1=10**

Q. 1. Fill in the blanks type questions :

(i) $U_t + U_{xxx} = 0$ is called _____.

(ii) Hamiltonian H is defined for _____.

(iii) The first form of Jacobi's theorem states that _____.

(iv) The relation between Poisson bracket and Lagrange bracket is _____.

(v) Hamiltonian canonical equations of motion for a conservative system are _____.

Multiple choice type questions :

(vi) Which of the following true for Poisson brackets ?

(a) They follow distributive law

(b) They follow commutative law

(c) Both (a) and (b) are true

(d) None of the above

(vii) The Legendre transform of L, $L^*(P)$ is given by :

(a) $\sup_{q \in R^n} \{p \cdot q + L(q)\}$

(b) $\sup_{q \in R^n} \{p \cdot q - L(q)\}$

(c) $\inf_{q \in R^n} \{p \cdot q + L(q)\}$

(d) $\inf_{q \in R^n} \{p \cdot q - L(q)\}$

(viii) $\frac{-d}{ds}(D_p\ L(\dot{X}(s), X(s)) + D_x L(\dot{X}(s), X(s)) = 0$

$\forall s \in [0, t]$ is called :

(a) Euler-Lagrange equation

(b) Hamilton-Jacobi equation

(c) Jacobi equation

(d) None of the above

(ix) The following is called wave equation :

(a) $|D_U| = 1$

(b) $U_t + UU_x = 0$

(c) $U_{tt} - \Delta U = 0$

(d) $U_t - \Delta U = 0$

(x) In a canonical transformation :

(a) $\dot{Q}_i = \frac{\partial K}{\partial P_i}, \dot{P}_i = \frac{-\partial K}{\partial Q_i}$

(b) $\dot{Q}_i = \frac{-\partial K}{\partial P_i}, \dot{P}_i = \frac{\partial K}{\partial Q_i}$

(c) $\dot{Q}_i = \frac{\partial K}{\partial Q_i}, \dot{P}_i = \frac{-\partial K}{\partial P_i}$

(d) $\dot{Q}_i = \frac{-\partial K}{\partial Q_i}, \dot{P}_i = \frac{\partial K}{\partial P_i}$

(xi) Out of the two statements :

(i) Lagrange's brackets are invariant under canonical transformation

(ii) Poisson brackets are invariant under canonical transformation

(a) (i) is true, (ii) is false

(b) (i) is false, (ii) is true

(c) (i) and (ii) both are true

(d) (i) and (ii) both are false

(xii) If U, V $\in L^2(R^n)$, then which one of will hold ?

(a) $\int_{R^n} U\overline{V}\,dx = \int_{R^n} \hat{U}\overline{\hat{V}}\,dy$

(b) $D^{\alpha}U = (iy)^{\alpha}\hat{U}$ for each multi index α

(c) $(\hat{U})^{v} = U$

(d) All of the above

SECTION - B

Note : Attempt any five questions. Each question carries 2 marks. **5×2=10**

Q. 2. Very short answer type questions (25-30 words) :

(i) Define Hamilton canonical equations.

(ii) State the second form of Jacobi theorem.

(iii) Explain Laplace method.

(iv) Define Potential functions.

(v) Explain Non-holonomic system.

(vi) Explain Poisson's bracket.

(vii) Define plane and travelling wave under similarity solution.

SECTION - C

Note : Attempt any five questions. Each question carries 4 marks. **5×4=20**

Q. 3. Short answer type questions (250 words) :

(i) Derive Hamilton principle form Lagrange's equation.

(ii) State and prove first form of Jacobi theorem.

(iii) Verify that the following transformation is canonical transformation :

$$P = \frac{1}{2}\left(p^2 + q^2\right), Q = \tan^{-1}\left(\frac{q}{p}\right)$$

(iv) Write short notes of Hodograph transforms.

(v) Derive stationary phase for wave equation.

(vi) Prove that Lagrange's brackets are invariant under canonical transformation.

(vii) State and prove properties of Fourier transform.

SECTION - D

Note : Attempt any three questions. Each question carries 10 marks. **3×10=30**

Q. 4. Essay type questions (more than 500 words) :

(i) Apply separation of variable technique to discover a solution of the porous medium equation $U_t - \Delta(U^r) = 0$ in $R^n X (0, \infty)$ where $U \geq 0$ and $r > 1$ constant.

(ii) State and prove Donkin's theorem.

(iii) Using Hopf-Cole transform, solve the quasilinear parabolic equation :

$U_t - a \Delta U + b |DU|^2 = 0$ in $R^4 \times (0, \infty)$ and

$U = g$ on $R^4 \times \{t = 0\}$

(iv) Explain : "The Euler's dynamical equation for the motion of a rigid body about an axis."