I-1051

M.A./M.Sc. (Final) Examination, 2020

MATHEMATICS

Paper - I

(Integration Theory & Functional Analysis)

Time Allowed : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 36

Note : Attempt any five questions. All questions carry equal marks.

Q. 1. State and prove Lebesgue decomposition theorem.

Q. 2. State and prove Radon-Nikodym theorem.

Q. 3. (a) Let n be a normed linear space and let x, y $\in$ N. Then prove that $\left| ||x|| - ||y|| \right| \le \|x - y\|$.

(b) Let f, g $\in$ Lp, where $1 \le p < \infty$, then prove that $||f + g||_p \le ||f||_p + ||g||_p$.

Q. 4. Let M be a closed linear subspace in a normed linear space N. For each coset x + M in the quotient space N/M we define :

$$\|x + M\| = \inf\left\{\|x + M\| : m \in M\right\}$$

Then prove that ||x + M|| is a norm on N/M and thus N/M is a normed linear space. Further if N is a Banach space, then so is N/M.

Q. 5. (a) Let N be a normed linear space and M a subspace of N. Then prove that the closure $\overline{M}$ of M is also a subspace of N.

(b) Let M be a closed linear subspace of a normed linear space N and let ϕ be the natural mapping of N onto N/M defined by $\phi(x) = x + M$. Show that ϕ is a continuous linear transformation for which $||\phi|| \le 1$.

Q. 6. State and prove Hahn Banach theorem.

Q. 7. State and prove closed graph theorem.

Q. 8. Let T be an operator on a normed linear space N, then its conjugate T* defined by $T^* : N^* \to N^* : T^*(f) = foT$ and $[T^*(f)](x) = f(T(x))$, for all $f \in N^*$ and all $x \in N$, is an operator on N* and the mapping $\phi : B(N) \to B(N^*) : \phi(T) = T^* \ \forall \ T \in B(N)$ is an isometric isomorphism of B(N) into B(N*) which reverse product and preserves the identity transformation.

Q. 9. (a) If x and y are any two vectors in Hilbert space, then prove that $||x + y||^2 + ||x - y||^2 = 2||x||^2 + 2||y||^2$.

(b) Let S be a non-empty subset of a Hilbert space H. Then prove that S^1 is a closed linear subspace of H.

Q. 10. State and prove Riesz representation theorem for continuous linear functional on a Hilbert space.